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Research on Quantum Effects of Coupled Double Resonance Circuit Using the Scheme of Gauss Propagator

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Abstract The time evolvement of quantum state has been researched by the schemes of Louisell's common canonical quantization and Gauss Propagator for mesoscopic coupled double resonance circuit with electrical sources. Moreover, the formulae of time evolvement wave function and transition probability of quantum state of this circuit are given. It is more significant for the quantitative analysis quantum character of mesoscopic circuit.

Keywords Gauss propagator · Mesoscopic circuit · Quantum state

1 Introduction

With the development of nanometer techniques and microelectronics, the trend of the miniaturization of integrated circuits and electronic components becomes stronger and stronger. Clearly, when the phase coherence length of the charge-carrier reaches the Fermi wavelength, quantum effects in electronic devices and circuit should be taken into account. In recent years, a lot of literatures [1–12] (Li and Chen 1996; Kyu et al. 1994; Li and Xu 2005; Li et al. 2006; Xu 2007; Xu et al. 2006, 2007; Chen et al. 1996; Wang and Sun 1997) indicated that quantum effects and quantum fluctuations were studied widely in vacuum state and squeezed vacuum state for mesoscopic circuit. But, time evolvement wave function and transition probability of quantum state have not been published yet for mesoscopic circuit with electrical sources. In this paper, depending on method of [13, 14] (Peng 1980; Zhu

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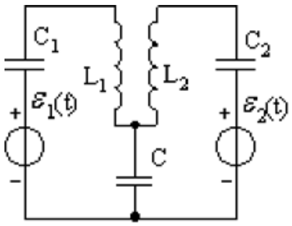

Fig. 1 Coupled double resonance circuit

1981) and basing on [15–21] (Zhang 1998; Cui 1998; Ling and Feng 1998; Feng 1997; Negro and Tartaglia 1981; Abdalla 1986; Oh et al. 1989), time evolvement of quantum state has been studied by the schemes of Louisell's common canonical quantization and Gauss propagator for mesoscopic coupled double resonance circuit. Moreover, the formulae of time evolvement wave function and transition probability of quantum state of this circuit are given. It is more significant for quantitative analysis quantum character of mesoscopic circuit.

2 The Scheme of Quantization

The circuit of Fig. 1 is coupled double resonance circuit with electric sources. The classical equations of motion of the system are

$$\dot{q}_1 = \frac{\phi_1}{L_1}, \qquad \dot{q}_2 = \frac{\phi_2}{L_2}, \tag{1}$$

$$\begin{aligned} \dot{\phi}_1 &= -A\left(\frac{1}{C_I} - \frac{m}{L_2 C}\right)q_1 - A\left(\frac{1}{C} - \frac{m}{L_2 C_{II}}\right)q_2 + A\varepsilon_1(t) - \frac{mA}{L_2}\varepsilon_2(t), \\ \dot{\phi}_2 &= -A\left(\frac{1}{C_{II}} - \frac{m}{L_1 C}\right)q_2 - A\left(\frac{1}{C} - \frac{m}{L_1 C_I}\right)q_1 + A\varepsilon_2(t) - \frac{mA}{L_1}\varepsilon_1(t), \end{aligned} \tag{2}$$

where charge q_u and magnetic flux $\phi_u = L_u \dot{q}_u$ $(u = 1, 2)$ denote generalized coordinate and generalized momentum, respectively. $\varepsilon_u(t)$ and m denote electrical source and mutual inductance, respectively

$$A = \frac{L_1 L_2}{L_1 L_2 - m^2}, \qquad \frac{1}{C_I} = \frac{1}{C_1} + \frac{1}{C}, \qquad \frac{1}{C_{II}} = \frac{1}{C_2} + \frac{1}{C}.$$

When $m = 0$, circuit has no coupling. When $m = L_1 L_2$, circuit can be regarded as a bizarre system by restrict dynamics theory of Dirac. Here, we consider $0 < m < \sqrt{L_1 L_2}$ only.

From (2), we obtain

$$\begin{aligned} \dot{\phi}_1 + \lambda\dot{\phi}_2 &= L_1\ddot{q}_1 + \lambda L_2 \ddot{q}_2 \\ &= -A\left[\frac{1}{C_I} - \frac{m}{L_2 C} + \lambda\left(\frac{1}{C} - \frac{m}{L_1 C_I}\right)\right]q_1 \\ &\quad - A\left[\lambda\left(\frac{1}{C_{II}} - \frac{m}{L_1 C}\right) + \frac{1}{C} - \frac{m}{L_2 C_{II}}\right]q_2 + f_1(t), \end{aligned} \tag{3}$$

where

$$f_1(t) = A\left(1 - \lambda\frac{m}{L_1}\right)\varepsilon_1(t) + A\left(\lambda - \frac{m}{L_2}\right)\varepsilon_2(t).$$

Letting

$$\frac{L_1}{\lambda L_2}=\frac{\frac{1}{C_I}-\frac{m}{L_2C}+\lambda(\frac{1}{C}-\frac{m}{L_1C_I})}{\lambda(\frac{1}{C_{II}}-\frac{m}{L_1C})+\frac{1}{C}-\frac{m}{L_2C_{II}}},$$

we can obtain

$$\lambda_1=\frac{\frac{L_1}{C_{II}}-\frac{L_2}{C_I}-(\frac{L_1}{C_{II}}-\frac{L_2}{C_I})\sqrt{1+4L_1L_2(\frac{1}{C}-\frac{m}{L_1C_I})(\frac{1}{C}-\frac{m}{L_2C_{II}})/(\frac{L_1}{C_{II}}-\frac{L_2}{C_I})^2}}{2L_2(\frac{1}{C}-\frac{m}{L_1C_I})}. \tag{4}$$

First, we consider the following coordinate transformations

$$Q_1=q_1+\lambda_1\frac{L_2}{L_1}q_2,\qquad p_1=\phi_1+\lambda_1\phi_2=L_1\dot{Q}_1. \tag{5}$$

From (3), we obtain

$$\dot{p}_1=-A\left[\frac{1}{C_I}-\frac{m}{L_2C}+\lambda_1\left(\frac{1}{C}-\frac{m}{L_1C_I}\right)\right]Q_1+f_1(t), \tag{6}$$

where

$$f_1(t)=A\left(1-\lambda_1\frac{m}{L_1}\right)\varepsilon_1(t)+A\left(\lambda_1-\frac{m}{L_2}\right)\varepsilon_2(t).$$

Similarly, from (2), we obtain

$$\begin{aligned}\lambda\dot{\phi}_1+\dot{\phi}_2&=\lambda L_1\ddot{q}_1+L_2\ddot{q}_2\\&=-A\left[\frac{1}{C_{II}}-\frac{m}{L_1C}+\lambda\left(\frac{1}{C}-\frac{m}{L_2C_{II}}\right)\right]q_2\\&\quad-\left[\lambda\left(\frac{1}{C_I}-\frac{m}{L_2C}\right)+\frac{1}{C}-\frac{m}{L_1C_I}\right]q_1+f_2(t),\end{aligned} \tag{7}$$

where

$$f_2(t)=A\left(1-\lambda\frac{m}{L_2}\right)\varepsilon_2(t)+A\left(\lambda-\frac{m}{L_1}\right)\varepsilon_1(t).$$

Letting

$$\frac{\lambda L_1}{L_2}=\frac{\lambda(\frac{1}{C_I}-\frac{m}{L_2C})+\frac{1}{C}-\frac{m}{L_1C_I}}{\frac{1}{C_{II}}-\frac{m}{L_1C}+\lambda(\frac{1}{C}-\frac{m}{L_2C_{II}})},$$

we can obtain

$$\lambda_2=\frac{\frac{L_2}{C_I}-\frac{L_1}{C_{II}}-(\frac{L_2}{C_I}-\frac{L_1}{C_{II}})\sqrt{1+4L_1L_2(\frac{1}{C}-\frac{m}{L_1C_I})(\frac{1}{C}-\frac{m}{L_2C_{II}})/(\frac{L_1}{C_{II}}-\frac{L_2}{C_I})^2}}{2L_1(\frac{1}{C}-\frac{m}{L_2C_{II}})}. \tag{8}$$

Second, we consider the following coordinate transformations

$$Q_2=\lambda_2\frac{L_1}{L_2}q_1+q_2,\qquad p_2=\lambda_2\phi_1+\phi_2=L_2\dot{Q}_2. \tag{9}$$

From (7), we can obtain

$$\dot{p}_2 = -A\left[\frac{1}{C_{II}} - \frac{m}{L_1 C} + \lambda_2\left(\frac{1}{C} - \frac{m}{L_2 C_{II}}\right)\right] Q_2 + f_2(t) \tag{10}$$

where

$$f_2(t) = A(1-\lambda_2 m/L_2)\varepsilon_2(t) + A(\lambda_2 - m/L_1)\varepsilon_1(t).$$

According to the scheme of Louisell's common canonical quantization, Q_u, P_u $(u = 1, 2)$ are regarded as twain canonical conjugate quantity. From canonical equations

$$\dot{Q}_u = \frac{\partial H}{\partial p_u}, \qquad \dot{p}_u = -\frac{\partial H}{\partial Q_u} \quad (u = 1, 2), \tag{11}$$

the Hamiltonian of this system can be written as

$$H = \frac{p_1^2}{2L_1} + \frac{p_2^2}{2L_2} + \frac{1}{2}L_1\omega_1^2 Q_1^2 + \frac{1}{2}L_2\omega_2^2 Q_2^2 - Q_1 f_1(t) - Q_2 f_2(t), \tag{12}$$

where

$$\omega_1 = \sqrt{A\left[\frac{1}{L_1 C_I} - \frac{m}{L_1 L_2 C} + \lambda_1\left(\frac{1}{L_1 C} - \frac{m}{L_1^2 C_I}\right)\right]},$$

$$\omega_2 = \sqrt{A\left[\frac{1}{L_2 C_{II}} - \frac{m}{L_1 L_2 C} + \lambda_2\left(\frac{1}{L_2 C} - \frac{m}{L_2^2 C_{II}}\right)\right]}.$$

When mechanics quantities are regarded as operators in Q-representation (namely, $\hat{Q}_u = Q_u \hat{p}_u = -i\hbar\frac{\partial}{\partial Q_u}$), the Hamiltonian of this system is

$$\hat{H} = -\frac{\hbar^2}{2L_1}\frac{\partial^2}{\partial Q_1^2} - \frac{\hbar^2}{2L_2}\frac{\partial^2}{\partial Q_2^2} + \frac{1}{2}L_1\omega_1^2 Q_1^2 + \frac{1}{2}L_2\omega_2^2 Q_2^2 - Q_1 f_1(t) - Q_2 f_2(t). \tag{13}$$

3 Gauss Propagator

We think that Gauss propagator is

$$\begin{aligned} &K(Q_1, Q_2, t; Q_{10}, Q_{20}, 0) \\ &\quad = A_0 \exp(-A_{11}Q_1^2 - A_{12}Q_1 - A_{13}Q_{10}^2 - A_{14}Q_{10} - A_{15}Q_1 Q_{10}) \\ &\qquad \times \exp(-A_{21}Q_2^2 - A_{22}Q_2 - A_{23}Q_{20}^2 - A_{24}Q_{20} - A_{25}Q_2 Q_{20}), \end{aligned} \tag{14}$$

where $A_0, A_{u1}, A_{u2}, A_{u3}, A_{u4}, A_{u5}$ $(u = 1, 2)$ denote pending coefficients, respectively, which all are functions of time t. Propagator K satisfies the following wave equation

$$i\hbar\frac{\partial}{\partial t}K = \hat{H}K. \tag{15}$$

Substituting (13) and (14) into (15), we obtain

$$i\hbar\dot{A}_{11} = \frac{2\hbar^2}{L_1}A_{11}^2 - \frac{1}{2}L_1\omega_1^2, \qquad i\hbar\dot{A}_{21} = \frac{2\hbar^2}{L_2}A_{21}^2 - \frac{1}{2}L_2\omega_2^2, \tag{16}$$

$$i\hbar\dot{A}_{12}=\frac{2\hbar^2}{L_1}A_{11}A_{12}+f_1(t),\qquad i\hbar\dot{A}_{22}=\frac{2\hbar^2}{L_2}A_{21}A_{22}+f_2(t),\tag{17}$$

$$i\hbar\dot{A}_{13}=\frac{\hbar^2}{2L_1}A_{15}^2,\qquad i\hbar\dot{A}_{23}=\frac{\hbar^2}{2L_2}A_{25}^2,\tag{18}$$

$$i\hbar\dot{A}_{14}=\frac{\hbar^2}{L_1}A_{12}A_{15},\qquad i\hbar\dot{A}_{24}=\frac{\hbar^2}{L_2}A_{22}A_{25},\tag{19}$$

$$i\hbar\dot{A}_{15}=\frac{2\hbar^2}{L_1}A_{11}A_{15},\qquad i\hbar\dot{A}_{25}=\frac{2\hbar^2}{L_2}A_{21}A_{25},\tag{20}$$

$$i\hbar\dot{A}_0=-\frac{\hbar^2}{2L_1}A_0A_{12}^2-\frac{\hbar^2}{2L_2}A_0A_{22}^2+\frac{\hbar^2}{L_1}A_0A_{11}+\frac{\hbar^2}{L_2}A_0A_{21}.\tag{21}$$

When $K(Q_1,Q_2,0;Q_{10},Q_{20},0)=\delta(Q_1-Q_{10},Q_2-Q_{20})$, from (16–20) and (21), we obtain

$$A_{11}=\frac{\omega_1L_1}{2i\hbar}\cot(\omega_1t),\qquad A_{21}=\frac{\omega_2L_2}{2i\hbar}\cot(\omega_2t),\tag{22}$$

$$\begin{aligned}A_{12}&=\frac{1}{i\hbar\sin(\omega_1t)}\int_0^t f_1(t')\cdot\sin(\omega_1t')dt',\\A_{22}&=\frac{1}{i\hbar\sin(\omega_2t)}\int_0^t f_2(t')\cdot\sin(\omega_2t')dt',\end{aligned}\tag{23}$$

$$A_{13}=\frac{\omega_1L_1}{2i\hbar}\cot(\omega_1t),\qquad A_{23}=\frac{\omega_2L_2}{2i\hbar}\cot(\omega_2t),\tag{24}$$

$$\begin{aligned}A_{14}&=\frac{\omega_1}{i\hbar}\int_0^t\frac{1}{\sin^2(\omega_1t')}[f_1(t)\cdot\sin(\omega_1t'')dt'']dt',\\A_{24}&=\frac{\omega_2}{i\hbar}\int_0^t\frac{1}{\sin^2(\omega_2t')}[f_2(t)\cdot\sin(\omega_2t'')dt'']dt',\end{aligned}\tag{25}$$

$$A_{15}=-\frac{\omega_1L_1}{i\hbar\cdot\sin(\omega_1t)},\qquad A_{25}=-\frac{\omega_2L_2}{i\hbar\cdot\sin(\omega_2t)},\tag{26}$$

$$\begin{aligned}A_0=&\left(\frac{\omega_1L_1}{2\pi i\hbar\cdot\sin(\omega_1t)}\right)^{1/2}\exp\left\{\frac{L_1}{2i\hbar}\int_0^{\mathrm{t}}\frac{1}{\sin^2(\omega_1\mathrm{t}')}\left[\int_0^{\mathrm{t}'}f_1(\mathrm{t})\cdot\sin(\omega_1\mathrm{t}'')dt''\right]^2dt'\right\}\\&\times\left(\frac{\omega_2L_2}{2\pi i\hbar\cdot\sin(\omega_2t)}\right)^{1/2}\exp\left\{\frac{L_2}{2i\hbar}\int_0^{\mathrm{t}}\frac{1}{\sin^2(\omega_2\mathrm{t}')}\left[\int_0^{\mathrm{t}'}f_2(\mathrm{t})\cdot\sin(\omega_2\mathrm{t}'')dt''\right]^2dt'\right\}.\end{aligned}\tag{27}$$

Therefore, Propagator K is

$$\begin{aligned}&K(Q_1,Q_2,t;Q_{10},Q_{20},0)\\&\quad=\left(\frac{\omega_1L_1}{2\pi i\hbar\cdot\sin(\omega_1t)}\right)^{1/2}\exp\left\{-A_{11}Q_1^2-A_{12}Q_1-A_{13}Q_{10}^2-A_{14}Q_{10}-A_{15}Q_1Q_{10}\right.\\&\qquad\left.+\frac{L_1}{2i\hbar}\int_0^t\frac{1}{\sin^2(\omega_1t')}\left[\int_0^{t'}f_1(t'')\cdot\sin(\omega_1t'')dt''\right]^2dt'\right\}\end{aligned}$$

$$\times\left(\frac{\omega_2 L_2}{2\pi i\hbar\cdot\sin(\omega_2 t)}\right)^{1/2}\exp\left\{-A_{21}Q_2^2-A_{22}Q_2-A_{23}Q_{20}^2-A_{24}Q_{20}-A_{25}Q_2Q_{20}\right.$$
$$\left.+\frac{L_2}{2i\hbar}\int_0^t\frac{1}{\sin^2(\omega_2 t')}\left[\int_0^{t'}f_2(t'')\cdot\sin(\omega_2 t'')dt''\right]^2dt'\right\}. \tag{28}$$

4 Wave Function

According to Propagator, wave function of this system is

$$\Psi(Q_1,Q_2,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dQ_{10}dQ_{20}K(Q_1,Q_2,t;Q_{10},Q_{20},0)\Psi_n(Q_{10},0)\Psi_m(Q_{20},0), \tag{29}$$

where

$$\begin{aligned}\Psi_n(Q_{10},0)&=N_n\cdot\exp\left(-\frac{\omega_1 L_1}{2\hbar}Q_{10}^2\right)\cdot H_n\left(\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_{10}\right),\\ \Psi_m(Q_{20},0)&=N_m\cdot\exp\left(-\frac{\omega_2 L_2}{2\hbar}Q_{20}^2\right)\cdot H_m\left(\sqrt{\frac{\omega_2 L_2}{\hbar}}Q_{20}\right).\end{aligned} \tag{30}$$

The eigenstate wave function is decided by $\hat{H}_0^u=-\frac{\hbar^2}{2L_u}\frac{\partial^2}{\partial Q_u^2}+\frac{1}{2}L_u\omega_u^2Q_u^2$ $(u=1,2)$, namely, original state wave function without electric source $(t<0)$. $H_n(\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_{10})$ and $H_m(\sqrt{\frac{\omega_2 L_2}{\hbar}}Q_{20})$ are Hermite polynomials. $N_n=(\frac{\sqrt{\omega_1 L_1/\hbar}}{\sqrt{\pi}2^n\cdot n!})^{1/2}$ and $N_m=(\frac{\sqrt{\omega_2 L_2/\hbar}}{\sqrt{\pi}2^m\cdot m!})^{1/2}$ are normalization constant.

Substituting (28), (30) into (29), we obtain

$$\Psi(Q_1,Q_2,t)=N_nI_n\exp(B_{11}Q_1^2+B_{12}Q_1+B_{13})\cdot N_mI_m\exp(B_{21}Q_2^2+B_{22}Q_2+B_{23}), \tag{31}$$

where

$$\begin{aligned}I_n&=\left[\frac{\omega_1 L_1}{2\pi i\hbar\cdot\sin(\omega_1 t)}\right]^{1/2}\int_{-\infty}^{\infty}\exp\left\{-\frac{\omega_1 L_1}{2\hbar}[1-i\cot(\omega_1 t)]\left[Q_{10}-\frac{Q_1-F_1(t)}{\exp(i\omega_1 t)}\right]\right\}\\ &\quad\times H_n\left(\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_{10}\right)dQ_{10},\\ I_m&=\left[\frac{\omega_2 L_2}{2\pi i\hbar\cdot\sin(\omega_2 t)}\right]^{1/2}\int_{-\infty}^{\infty}\exp\left\{-\frac{\omega_2 L_2}{2\hbar}[1-i\cot(\omega_2 t)]\left[Q_{20}-\frac{Q_2-F_2(t)}{\exp(i\omega_2 t)}\right]\right\}\\ &\quad\times H_m\left(\sqrt{\frac{\omega_2 L_2}{\hbar}}Q_{20}\right)dQ_{20},\end{aligned} \tag{32}$$

with

$$\begin{aligned}F_1(t)&=\frac{\sin(\omega_1 t)}{L_1}\int_0^t\frac{1}{\sin^2(\omega_1 t')}\left[\int_0^{t'}f_1(t'')\cdot\sin(\omega_1 t'')dt''\right]dt',\\ F_2(t)&=\frac{\sin(\omega_2 t)}{L_2}\int_0^t\frac{1}{\sin^2(\omega_2 t')}\left[\int_0^{t'}f_2(t'')\cdot\sin(\omega_2 t'')dt''\right]dt',\end{aligned} \tag{33}$$

$$
\begin{aligned}
B_{11} &= \frac{1}{4}\frac{A_{15}^2}{\frac{\omega_1 L_1}{2\hbar}+A_{13}} - A_{11} = -\frac{\omega_1 L_1}{2\hbar}, \qquad B_{12} = \frac{1}{4}\frac{2A_{14}A_{15}}{\frac{\omega_1 L_1}{2\hbar}+A_{13}} - A_{12},\\
B_{13} &= \frac{1}{4}\frac{A_{14}^2}{\frac{\omega_1 L_1}{2\hbar}+A_{13}} + \frac{1}{2i\hbar}\int_0^t \frac{1}{\sin^2(\omega_1 t')}\left[\int_0^{t'} f_1(t'')\cdot\sin(\omega_1 t'')dt''\right]dt',\\
B_{21} &= \frac{1}{4}\frac{A_{25}^2}{\frac{\omega_2 L_2}{2\hbar}+A_{23}} - A_{21} = -\frac{\omega_2 L_2}{2\hbar}, \qquad B_{22} = \frac{1}{4}\frac{2A_{24}A_{25}}{\frac{\omega_2 L_2}{2\hbar}+A_{23}} - A_{22},\\
B_{23} &= \frac{1}{4}\frac{A_{24}^2}{\frac{\omega_2 L_2}{2\hbar}+A_{23}} + \frac{1}{2i\hbar}\int_0^t \frac{1}{\sin^2(\omega_2 t')}\left[\int_0^{t'} f_2(t'')\cdot\sin(\omega_2 t'')dt''\right]dt'.
\end{aligned} \tag{34}
$$

Using the following formulas

$$
\exp(2\tau\xi - \tau^2) = \sum_{n=0}^{\infty} H_n(\xi)\cdot\frac{\tau^n}{n!}, \tag{35}
$$

$$
\int_{-\infty}^{\infty}\exp[-\alpha(x-\beta)^2+\gamma x+\eta]dx = \sqrt{\frac{\pi}{\alpha}}\cdot\exp\left(\beta\gamma+\frac{\gamma^2}{4\alpha}+\eta\right), \quad \mathrm{Re}(\alpha)>0, \tag{36}
$$

we obtain

$$
\begin{aligned}
\sum_{n=0}^{\infty} I_n\cdot\frac{\tau^n}{n!} &= \left[\frac{\omega_1 L_1}{2\pi i\hbar\sin(\omega_1 t)}\right]^{1/2}\int_{-\infty}^{\infty}\exp\left\{-\frac{\omega_1 L_1}{2\hbar}[1-i\cot(\omega_1 t)]\left[Q_{10}-\frac{Q_1-F_1(t)}{\exp(i\omega_1 t)}\right]^2\right\}\\
&\quad\times\left[\sum_{n=0}^{\infty} H_n\left(\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_{10}\right)\cdot\frac{\tau^n}{n!}\right]dQ_{10}\\
&= \left[\frac{\omega_1 L_1}{2\pi i\hbar\sin(\omega_1 t)}\right]^{\frac{1}{2}}\int_{-\infty}^{\infty}\exp\left\{-\frac{\omega_1 L_1}{2\hbar}\frac{\exp(i\omega_1 t)}{i\sin(\omega_1 t)}\left[Q_{10}-\frac{Q_1-F_1(t)}{\exp(i\omega_1 t)}\right]^2\right.\\
&\quad\left.+2\tau\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_{10}-\tau^2\right\}dQ_{10}\\
&= \left[\frac{\omega_1 L_1}{2\pi i\hbar\sin(\omega_1 t)}\right]^{\frac{1}{2}}\left[\frac{2\pi i\hbar\sin(\omega_1 t)}{\exp(i\omega_1 t)}\right]^{\frac{1}{2}}\exp\left\{2\sqrt{\frac{\omega_1 L_1}{\hbar}}[Q_1-F_1(t)]\right.\\
&\quad\left.\times[\tau\cdot\exp(-i\omega_1 t)]-[\tau\cdot\exp(-i\omega_1 t)]^2\right\}\\
&= \exp\left(-i\frac{1}{2}\omega_1 t\right)\sum_{n=0}^{\infty} H_n\left[\sqrt{\frac{\omega_1 L_1}{\hbar}}(Q_1-F_1(t))\right]\cdot\frac{[\tau\cdot\exp(-i\omega_1 t)]^n}{n!}\\
&= \sum_{n=0}^{\infty}\exp\left[-i\left(n+\frac{1}{2}\right)\Omega t\right]\cdot H_n\left[\sqrt{\frac{\Omega L}{\hbar}}(Q-F(t))\right]\cdot\frac{\tau^n}{n!}.
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
I_n &= \exp\left[-i\left(n+\frac{1}{2}\right)\omega_1 t\right]\cdot H_n\left[\sqrt{\frac{\omega_1 L_1}{\hbar}}(Q_1-F_1(t))\right],\\
I_m &= \exp\left[-i\left(m+\frac{1}{2}\right)\omega_2 t\right]\cdot H_m\left[\sqrt{\frac{\omega_2 L_2}{\hbar}}(Q_2-F_2(t))\right].
\end{aligned} \tag{37}
$$

Substituting (37) into (31), the wave function of mesoscopic coupled double resonance circuit with electric source has the form

$$\Psi(Q_1,Q_2,t)=N_nH_n\left[\sqrt{\frac{\omega_1L_1}{\hbar}}(Q_1-F_1(t))\right]\exp\left(-\frac{\omega_1L_1}{2\hbar}Q_1^2+B_{12}Q_1+B_{13}\right)$$
$$\times\exp\left[-i\left(n+\frac{1}{2}\right)\omega_1t\right]N_mH_m\left[\sqrt{\frac{\omega_2L_2}{\hbar}}(Q_2-F_2(t))\right]$$
$$\times\exp\left(-\frac{\omega_2L_2}{2\hbar}Q_2^2+B_{22}Q_2+B_{23}\right)\exp\left[-i\left(m+\frac{1}{2}\right)\omega_2t\right]. \quad (38)$$

5 Time Evolvement of Quantum State

Without electric source ($t<0$), the circuit is in ground state

$$\Psi_{00}(Q_1,Q_2)=\left(\frac{\sqrt{\omega_1L_1/\hbar}}{\sqrt{\pi}}\right)^{1/2}\cdot\exp\left(-\frac{\omega_1L_1}{2\hbar}Q_1^2\right)\cdot\left(\frac{\sqrt{\omega_2L_2/\hbar}}{\sqrt{\pi}}\right)^{1/2}\cdot\exp\left(-\frac{\omega_2L_2}{2\hbar}Q_2^2\right),$$

which is decided by

$$\hat{H}_0=-\frac{\hbar^2}{2L_1}\frac{\partial^2}{\partial Q_1^2}+\frac{1}{2}L_1\omega_1^2-\frac{\hbar^2}{2L_2}\frac{\partial^2}{\partial Q_2^2}+\frac{1}{2}L_2\omega_2^2.$$

Under electric source ($t\geq 0$), the wave function is

$$\Psi(Q_1,Q_2,t)=\left(\frac{\omega_1L_1}{\pi\hbar}\right)^{1/4}\cdot\exp\left(-\frac{\omega_1L_1}{2\hbar}Q_1^2+B_{12}Q_1+B_{13}\right)\cdot\exp\left(-i\frac{1}{2}\omega_1t\right)$$
$$\times\left(\frac{\omega_2L_2}{\pi\hbar}\right)^{1/4}\cdot\exp\left(-\frac{\omega_2L_2}{2\hbar}Q_2^2+B_{22}Q_2+B_{23}\right)\cdot\exp\left(-i\frac{1}{2}\omega_2t\right). \quad (39)$$

From (39), using eigenvector $\Psi_{nm}(Q_1,Q_2)=\Psi_n(Q_1)\Psi_m(Q_2)$ of $\hat{H}_0$ to expand $\Psi(Q_1,Q_2,t)=\sum_{nm}a_{nm}\Psi_{nm}(Q_1,Q_2)$, we obtain

$$a_{nm}(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi_{nm}^*(Q_1,Q_2)\cdot\Psi(Q,t)dQ_1dQ_2$$
$$=\left(\frac{\omega_1L_1}{\pi\hbar}\right)^{1/4}N_n\exp\left(-i\frac{1}{2}\omega_1t\right)$$
$$\times\int_{-\infty}^{\infty}\exp\left(-\frac{\omega_1L_1}{\hbar}Q_1^2+B_{12}+B_{13}\right)$$
$$\times H_n\left(\sqrt{\frac{\omega_1L_1}{\hbar}}Q_1\right)dQ_1\cdot\left(\frac{\omega_2L_2}{\pi\hbar}\right)^{1/4}N_m\exp\left(-i\frac{1}{2}\omega_2t\right)$$
$$\times\int_{-\infty}^{\infty}\exp\left(-\frac{\omega_2L_2}{\hbar}Q_2^2+B_{22}+B_{23}\right)\cdot H_m\left(\sqrt{\frac{\omega_2L_2}{\hbar}}Q_2\right)dQ_2$$
$$=a_n(t)\cdot a_m(t), \quad (40)$$

where

$$a_n(t)=\left(\frac{\omega_1 L_1}{\pi\hbar}\right)^{1/4} N_n \exp\left(-i\frac{1}{2}\omega_1 t\right)\int_{-\infty}^{\infty}\exp\left(-\frac{\omega_1 L_1}{\hbar}Q_1^2+B_{12}+B_{13}\right)\times H_n\left(\sqrt{\frac{\omega_1 L_1}{\hbar}}Q_1\right)dQ_1,$$

$$a_m(t)=\left(\frac{\omega_2 L_2}{\pi\hbar}\right)^{1/4} N_m \exp\left(-i\frac{1}{2}\omega_2 t\right)\int_{-\infty}^{\infty}\exp\left(-\frac{\omega_2 L_2}{\hbar}Q_2^2+B_{22}+B_{23}\right)\times H_m\left(\sqrt{\frac{\omega_2 L_2}{\hbar}}Q_2\right)dQ_2.$$

Using (28, 29) and $\exp(\tau\zeta)=\sum_{m=0}^{\infty}\zeta^m\cdot\frac{\tau^m}{m!}$ again, we have

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{a_n}{N_n}\cdot\frac{\tau^n}{n!}&=\left(\frac{\omega_1 L_1}{\pi\hbar}\right)^{1/4}\exp\left(-i\frac{1}{2}\omega_1 t\right)\int_{-\infty}^{\infty}\exp\left(-\frac{\omega_1 L_1}{\hbar}Q_1^2+B_{12}Q_1+B_{13}\right)\\&\quad\times\sum_{n=0}^{\infty}H_n\left(\sqrt{\frac{\omega_1 L_1}{\hbar}Q_1}\right)\cdot\frac{\tau^n}{n!}dQ_1\\&=\left(\frac{\Omega L}{\pi\hbar}\right)^{1/4}\cdot\exp\left(-i\frac{1}{2}\Omega t\right)\int_{-\infty}^{\infty}\exp\left[-\frac{\omega_1 L_1}{\hbar}Q_1^2+\left(B_{12}+2\tau\sqrt{\frac{\omega_1 L_1}{\hbar}}\right)Q_1\right.\\&\quad\left.+B_{13}-\tau^2\right]dQ_1\\&=\left(\frac{\pi\hbar}{\Omega L}\right)^{1/4}\exp\left(-i\frac{1}{2}\Omega t+\frac{\hbar}{4\Omega L}B_2^2+B_3\right)\exp\left(\frac{B_2}{\sqrt{\frac{\Omega L}{\hbar}}}\tau\right)\\&=\left(\frac{\pi\hbar}{\omega_1 L_1}\right)^{1/4}\exp\left(-i\frac{1}{2}\omega_1 t+\frac{\hbar}{4\omega_1 L_1}B_{12}^2+B_{13}\right)\sum_{m=0}^{\infty}\left(\sqrt{\frac{\hbar}{\omega_1 L_1}}B_{12}\right)^n\cdot\frac{\tau^n}{n!}.\end{aligned}$$

Therefore

$$\begin{aligned}\frac{a_n(t)}{N_n}&=\left(\frac{\pi\hbar}{\omega_1 L_1}\right)^{1/4}\exp\left(-i\frac{1}{2}\omega_1 t+\frac{\hbar}{4\omega_1 L_1}B_{12}^2+B_{13}\right)\left(\sqrt{\frac{\hbar}{\omega_1 L_1}}B_{12}\right)^n,\\a_n(t)&=\left(\sqrt{\frac{\hbar}{\omega_1 L_1}}B_{12}\right)^n(2^n n!)^{-\frac{1}{2}}\exp\left(-i\frac{1}{2}\omega_1 t+\frac{\hbar}{4\omega_1 L_1}B_{12}^2+B_{13}\right).\end{aligned}\tag{41}$$

In the same way, we have

$$a_m(t)=\left(\sqrt{\frac{\hbar}{\omega_2 L_2}}B_{12}\right)^m(2^m m!)^{-\frac{1}{2}}\exp\left(-i\frac{1}{2}\omega_2 t+\frac{\hbar}{4\omega_2 L_2}B_{22}^2+B_{23}\right).$$

So, we can obtain that transition probability from original state $\Psi_{00}(Q_1,Q_2)$ to excitation state $\Psi_{nm}(Q_1,Q_2)$ are

$$\begin{aligned}w_{nm} &= |a_{nm}(t)|^2\\&= \left(\frac{\hbar}{\omega_1 L_1}\right)^n \cdot \frac{(B_{12}B_{12}^*)^n}{2^n \cdot n!} \cdot \exp\left(\frac{\hbar}{4\omega_1 L_1}(B_{12}^2 + B_{12}^{*2}) + B_{13} + B_{13}^*\right)\\&\quad \times \left(\frac{\hbar}{\omega_2 L_2}\right)^m \cdot \frac{(B_{22}B_{22}^*)^m}{2^m \cdot m!} \cdot \exp\left(\frac{\hbar}{4\omega_2 L_2}(B_{22}^2 + B_{22}^{*2}) + B_{23} + B_{23}^*\right).\end{aligned} \tag{42}$$

6 Discussion

When $\varepsilon_1(t) = \varepsilon_2(t) = 0$, according to (23), (25) and (34), (38) becomes

$$\begin{aligned}\Psi(Q_1, Q_2, t) &= N_n \cdot \exp\left(-\frac{\omega_1 L_1}{2\hbar} Q_1^2\right) \cdot H_n\left(\sqrt{\frac{\omega_1 L_1}{2\hbar}} Q_1\right) \cdot \exp\left(-i\left(n + \frac{1}{2}\right)\omega_1 t\right)\\&\quad \times N_m \cdot \exp\left(-\frac{\omega_2 L_2}{2\hbar} Q_2^2\right) \cdot H_m\left(\sqrt{\frac{\omega_2 L_2}{2\hbar}} Q_2\right) \cdot \exp\left(-i\left(m + \frac{1}{2}\right)\omega_2 t\right)\end{aligned} \tag{43}$$

which is eigen wave function of harmonic oscillator. Equation (42) becomes

$$w_{nm} = 0, \tag{44}$$

namely, without interfere of electric source, without transition, the system is original stationary state at all time. It is accorded with stationary state theory of quantum mechanics. It is shown that equation (38) of wave function is more significant for mesoscopic RLC circuit with electric source.

When $t = 0$, $\varepsilon_1(t) = \varepsilon_2(t) = 0$, according to (5) and (9), we obtain the quantum fluctuations of charge q_1, q_2, magnetic flux ϕ_1, ϕ_2 and current j_1, j_2, and they are

$$\begin{aligned}\langle(\Delta q_1)^2\rangle &= \frac{\hbar}{2(1-\lambda_1\lambda_2)^2}\left[\frac{1}{\omega_1 L_1} + \lambda_1^2 \frac{L_2^2}{L_1^2}\frac{1}{\omega_2 L_2}\right],\\\langle(\Delta q_2)^2\rangle &= \frac{\hbar}{2(1-\lambda_1\lambda_2)^2}\left[\frac{1}{\omega_2 L_2} + \lambda_2^2 \frac{L_1^2}{L_2^2}\frac{1}{\omega_1 L_1}\right],\end{aligned} \tag{45}$$

$$\begin{aligned}\langle(\Delta \phi_1)^2\rangle &= \frac{\hbar}{2(1-\lambda_1\lambda_2)^2}[\omega_1 L_1 + \lambda_1^2 \omega_2 L_2],\\\langle(\Delta \phi_2)^2\rangle &= \frac{\hbar}{2(1-\lambda_1\lambda_2)^2}[\lambda_2^2 \omega_2 L_2 + \omega_1 L_1],\end{aligned} \tag{46}$$

$$\begin{aligned}\langle(\Delta j_1)^2\rangle &= \frac{\hbar}{2L_1^2(1-\lambda_1\lambda_2)^2}[\omega_1 L_1 + \lambda_1^2 \omega_2 L_2],\\\langle(\Delta j_2)^2\rangle &= \frac{\hbar}{2L_2^2(1-\lambda_1\lambda_2)^2}[\lambda_2^2 \omega_2 L_2 + \omega_1 L_1].\end{aligned} \tag{47}$$

We can see that, the quantum fluctuations of charge, magnetic flux and current depend on circuit device parameter for mesoscopic double coupling resonance circuit with electric source. So the quantum noise can be controled by adjusting circuit device parameters. Therefore, studying quantum effects of the mesoscopic circuit is more significant.

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